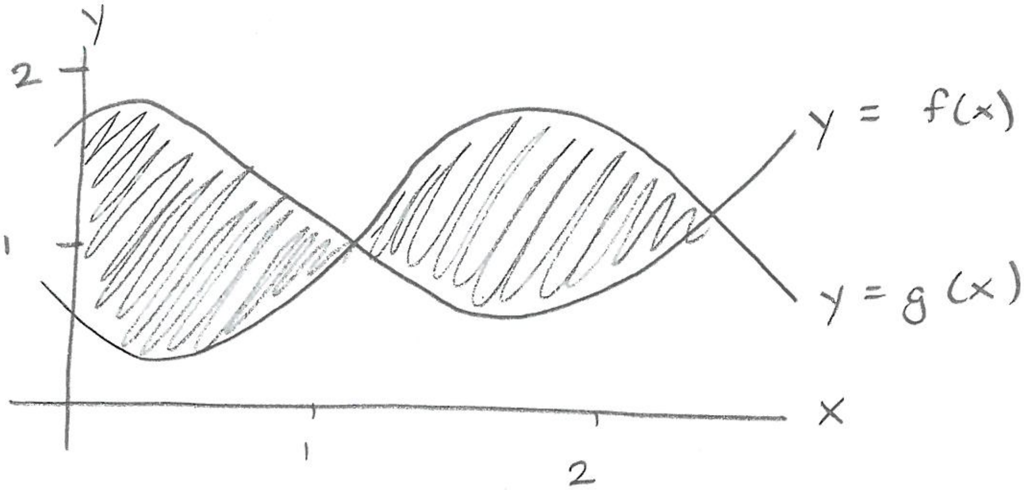
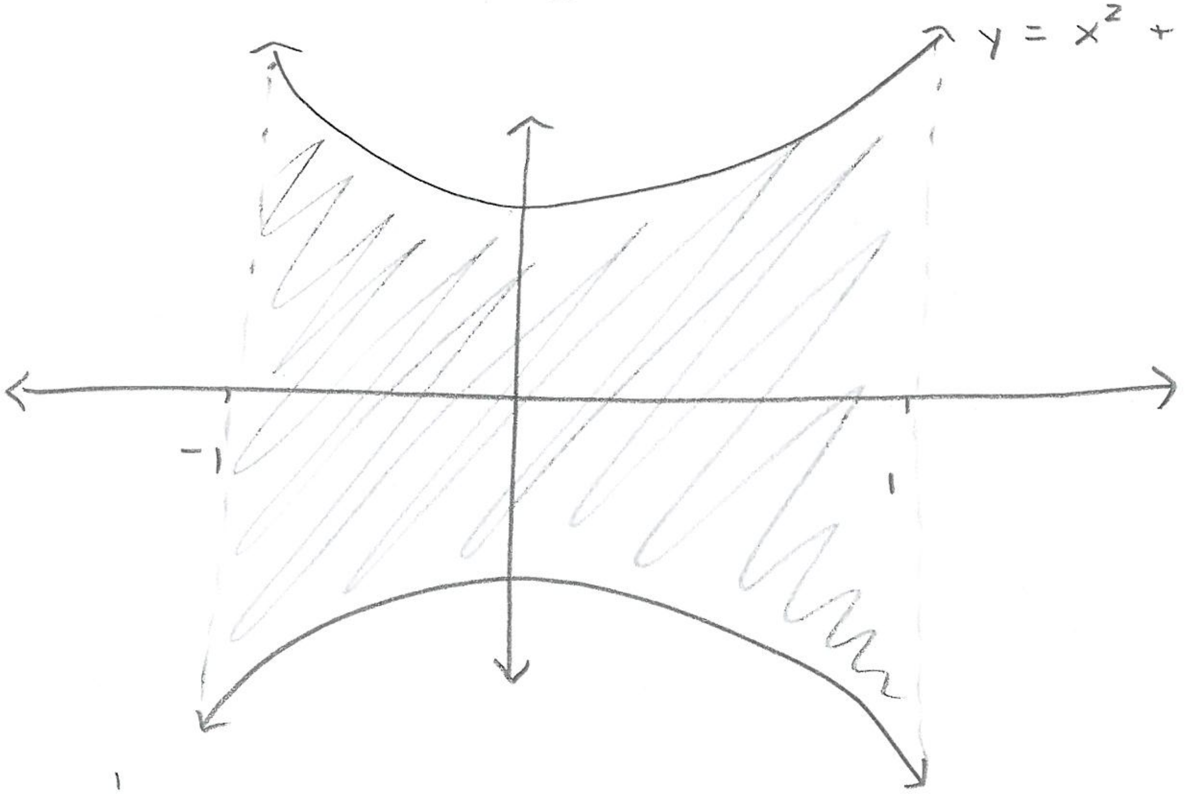
MA 131 HW #12

Section 6.4

4.)



6.)



$$A = \int_{-1}^{1} \left(x^2 + 1 - (-x^2 - 1) \right) dx$$

$$= 2\int_{-1}^{1} (x^2 + 1)\, dx$$

$$= 2\left(\frac{x^3}{3} + x \Big|_{-1}^{1} \right) = 2\left(\frac{1}{3} + 1 - \left(\frac{(-1)^3}{3} - 1 \right) \right) = 2\left(\frac{2}{3} + 2 \right) = \boxed{\frac{16}{3}}$$

11.) <u>Find intersections:</u> $x^2 = x$

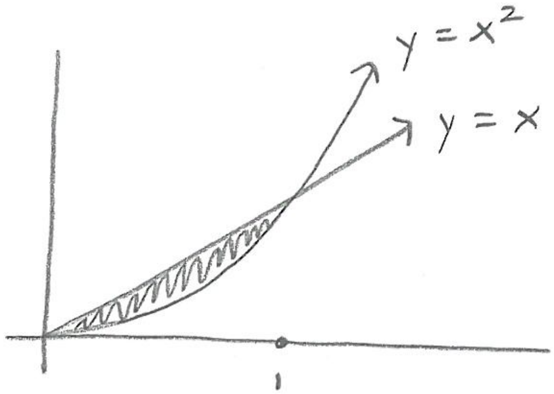
$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$\boxed{x = 0,\ x = 1}$$

<u>Test point:</u> Pick $x = 1/2$

$$\left(\tfrac{1}{2}\right)^2 = \tfrac{1}{4} < \tfrac{1}{2} \longrightarrow y = x \text{ is top}$$



$$A = \int_0^1 (x - x^2)\,dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

13.) $-x^2 + 6x - 5 = 2x - 5$

$$0 = x^2 - 4x$$

$$= x(x-4)$$

$$\boxed{x = 0,\ x = 4}$$

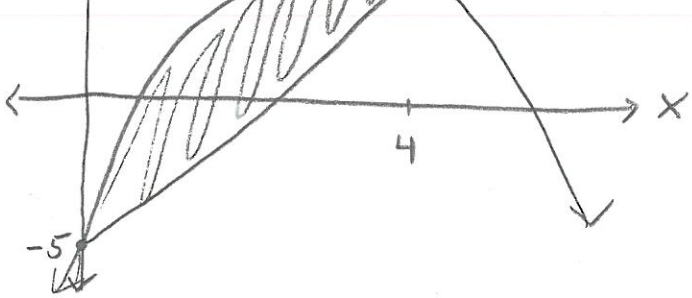
$$-(2)^2 + 6(2) - 5 = -4 + 12 - 5$$

$$= 3$$

$$2(2) - 5 = 4 - 5 = -1$$

$$3 > -1$$

so $\boxed{-x^2 + 6x - 5 \text{ is top}}$



$$A = \int_0^4 (-x^2 + 6x - 5 - 2x + 5)\,dx$$

$$= \int_0^4 (-x^2 + 4x)\,dx$$

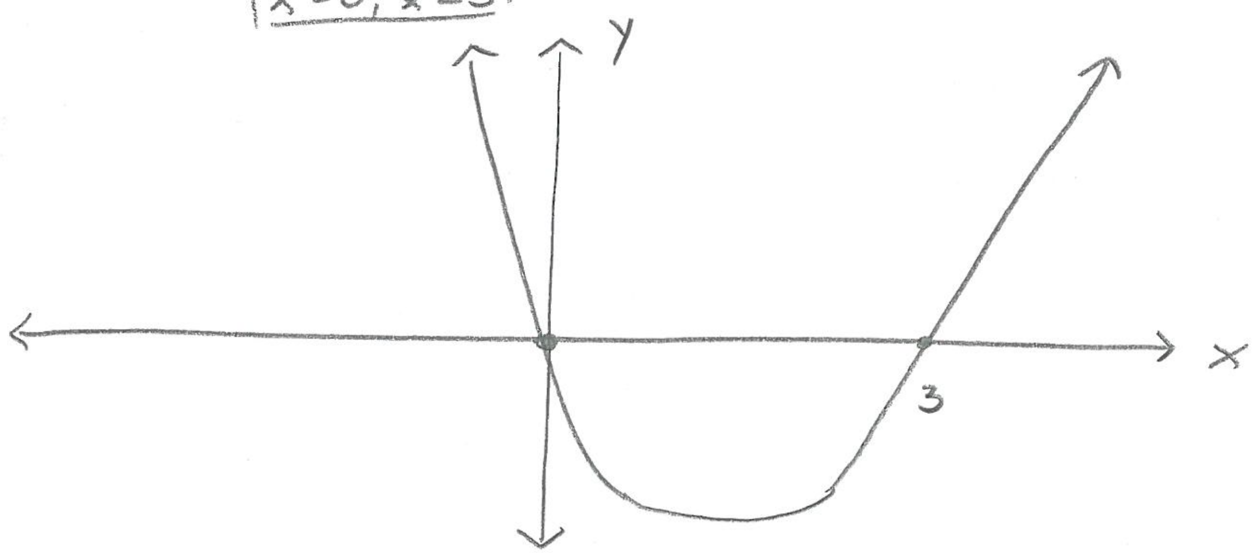
$$= -\frac{x^3}{3} + 2x^2 \Big|_0^4 = -\frac{64}{3} + 32 = \frac{-64 + 96}{3} = \boxed{\frac{32}{3}}$$

19.) First lets get an idea of what $y = x^2 - 3x$ looks like.

When does $y = 0$?

$$0 = x^2 - 3x = x(x-3)$$

$\boxed{x = 0,\ x = 3}$



a.) $$A = -\int_0^3 (x^2-3x)\,dx$$

$$= -\left(\frac{x^3}{3} - \frac{3x^2}{2}\Big|_0^3\right)$$

$$= -\left(\frac{27}{3} - \frac{27}{2}\right) = -\left(\frac{54-81}{6}\right) = \boxed{\frac{27}{6}} = \boxed{\frac{9}{2}}$$

b) $$A = -\int_0^3 (x^2-3x)\,dx + \int_3^4 (x^2-3x)\,dx$$

$$= \frac{27}{6} + \left(\frac{x^3}{3} - \frac{3x^2}{2}\Big|_3^4\right)$$

$$= \frac{27}{6} + \frac{4^3}{3} - \frac{48}{2} - \left(-\frac{27}{6}\right)$$

$$= \frac{54}{6} + \frac{64}{3} - \frac{48}{2} = \frac{54}{6} + \frac{128}{6} - \frac{144}{6} = \frac{38}{6} = \boxed{\frac{19}{3}}$$

c) $$A = \int_{-2}^0 (x^2-3x)\,dx + \frac{9}{2}$$

$$= \frac{x^3}{3} - \frac{3x^2}{2}\Big|_{-2}^0 + \frac{9}{2}$$

$$= -\left(\frac{(-2)^3}{3} - \frac{3(-2)^2}{2}\right) + \frac{9}{2} = -\left(\frac{-8}{3} - \frac{12}{2}\right) + \frac{9}{2}$$

$$= \frac{8}{3} + 6 + \frac{9}{2} = \frac{16+36+27}{6} = \boxed{\frac{79}{6}}$$

28.) A is the distance between the two rockets after 10 seconds

Section 6.5

1.) $\frac{1}{3-0}\int_0^3 x^2\,dx = \frac{1}{3}\left(\frac{x^3}{3}\Big|_0^3\right) = \frac{1}{3}\left(\frac{27}{3} - 0\right) = \boxed{3}$

4.) $\frac{1}{1-0}\int_0^1 2\,dx = 2x\Big|_0^1 = 2 - 0 = \boxed{2}$

7.) $\frac{1}{12}\int_0^{12}\left(47 + 4t - \frac{1}{3}t^2\right)dt = \frac{1}{12}\left(47t + 2t^2 - \frac{t^3}{9}\Big|_0^{12}\right)$

$= \frac{1}{12}\left(47(12) + 2(12)^2 - \frac{12^3}{9}\right) = \boxed{55^\circ}$

29.) $V = \pi\int_{-2}^{2}(4 - x^2)\,dx = \pi\left(4x - \frac{x^3}{3}\Big|_{-2}^{2}\right)$

$= \pi\left(4(2) - \frac{8}{3} - \left(4(-2) - \frac{(-2)^3}{3}\right)\right)$

$= \pi\left(8 - \frac{8}{3} + 8 - \frac{8}{3}\right)$

$= \pi\left(16 - \frac{16}{3}\right)$

$= \boxed{\pi\ 32/3}$

30.) $$V=\pi\int_{-r}^{r}(r^2-x^2)\,dx = \pi\left(r^2x-\frac{x^3}{3}\Big|_{-r}^{r}\right)$$

$$=\pi\left(r^3-\frac{r^3}{3}-\left(r^2(-r)-\frac{(-r)^3}{3}\right)\right)$$

$$=\pi\left(r^3-\frac{r^3}{3}+r^3-\frac{r^3}{3}\right)$$

$$=\pi\left(2r^3-\frac{2r^3}{3}\right)=\pi\left(\frac{6r^3-2r^3}{3}\right)=\boxed{\frac{4\pi r^3}{3}}$$

32.) $$V=\pi\int_0^h k^2x^2\,dx=\pi k^2\int_0^h x^2\,dx=\boxed{\pi k^2\frac{h^3}{3}}$$

Section 9.1

1.) $u=x^2+4$

$du=2x\,dx$

$$\int 2x(2x+4)^5\,dx=\int u^5\,du=\frac{u^6}{6}+C=\boxed{\frac{(x^2+4)^6}{6}+C}$$

2.) $u=2x-1$

$du=2\,dx$

$$\int 2(2x-1)^7\,dx=\int u^7\,du=\frac{u^8}{8}+C=\boxed{\frac{(2x-1)^8}{8}+C}$$

5.) $u = x^3 - 1$

$du = 3x^2 dx$

$$\int 3x^2 e^{x^3-1}\,dx = \int e^u\,du = e^u + C = \boxed{e^{x^3-1} + C}$$

11.) $u = x^2$

$du = 2x\,dx \longrightarrow \frac{1}{2}du = x\,dx$

$$\int x e^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{x^2} + C}$$

15.) $u = x^5 + 1$

$du = 5x^4 dx \longrightarrow \frac{1}{5}du = x^4 dx$

$$\int \frac{x^4}{x^5+1}\,dx = \int \frac{1}{u}\,du = \ln u + C = \boxed{\ln(x^5+1) + C}$$

Section 9.3

3.) $$\int_0^2 4x(1+x^2)^3\,dx = 2\int_1^5 u^3\,du$$

$\boxed{u = 1 + x^2}$

$du = 2x\,dx$

$\boxed{4x\,dx = 2du}$

$$= 2\left(\frac{u^4}{4}\Big|_1^5\right)$$

$$= 2\left(\frac{5^4}{4} - \frac{1}{4}\right)$$

$$= 2\left(\frac{625-1}{4}\right) = \frac{624}{2} = \boxed{312}$$

11.) $\boxed{u = x^2 + 3}$

$du = 2x\,dx$

$\boxed{\frac{1}{2}du = x\,dx}$

$$\int_0^1 \frac{x}{x^2+3}\,dx = \frac{1}{2}\int_3^4 \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln u \Big|_3^4$$

$$= \boxed{\frac{\ln 4 - \ln 3}{2}}$$

13.) $\boxed{u = x^3}$

$du = 3x^2\,dx$

$\boxed{\frac{1}{3}du = x^2\,dx}$

$$\int_1^3 x^2 e^{x^3}\,dx = \frac{1}{3}\int_1^{27} e^u\,du$$

$$= \frac{1}{3}e^u \Big|_1^{27}$$

$$= \boxed{\frac{e^{27} - e}{3}}$$