Homework #3 – MA131

Section 1.4

18.) This is a rational function, but we can't just plug in 1 since this would make the denominator 0

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1} (x+1) = 2$$

20.) Again, can't plug in $x=3$ here

$$\lim_{x\to 3} \frac{x^2-x-6}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+2)}{x-3} = \lim_{x\to 3} (x+2) = 5$$

49.) This is the definition of $\frac{d}{dx}x^2$ when $x=1$

$f(x) = x^2$

$a = 1$

$$\lim_{h\to 0} \frac{(1+h)^2-1}{h} = f'(1) = 2$$

50.) Like # 49

$f(x) = x^3$

$a = 2$

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h} = f'(2) = 3\cdot 2^2 = 3\cdot 4 = 12$$

Section 1.6

2.) $y' = 3 \cdot 4x^3 = \boxed{12x^3}$

4.) $y' = \frac{d}{dx}\left(\frac{1}{3x^3}\right) = \frac{1}{3}\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{1}{3}\frac{d}{dx}(x^{-3}) = -x^{-4} = \boxed{\frac{-1}{x^4}}$

6.) $\boxed{f'(x) = 0}$ since f is a constant

8.) $y' = 4 \cdot 3x^2 - 2 \cdot 2x + 1$

$= \boxed{12x^2 - 4x + 1}$

10.) $y' = 3(x^2-1)^2 \frac{d}{dx}(x^2-1)$

$= 3(x^2-1)^2(2x)$

$\boxed{= 6x(x^2-1)^2}$

12.) $y' = -2(x^2+x)^{-3} \frac{d}{dx}(x^2+x)$

$= -2(x^2+x)^{-3}(2x+1)$

$= \boxed{\frac{-4x-2}{(x^2+x)^3}}$

18.) $y' = 3(x-1)^2 \frac{d}{dx}(x-1) + 4(x+2)^3 \frac{d}{dx}(x+2)$

$= 3(x-1)^2(1) + 4(x+2)^3(1)$

$\boxed{= 3(x-1)^2 + 4(x+2)^3}$

36.) $\boxed{y' = \pi^2}$

58.a) Sales on Jan 10 $= S(10)$

$$= 3 + \frac{9}{11^2}$$

$$= 3 + \frac{9}{121}$$

$$\boxed{\approx \$3,074}$$

$$S'(x) = \frac{d}{dx}\left(3 + \frac{9}{(x+1)^2}\right)$$

$$= 0 + 9\frac{d}{dx}\left((x+1)^{-2}\right)$$

$$= 9(-2)(x+1)^{-3}\frac{d}{dx}(x+1)$$

$$= \frac{-18}{(x+1)^3}$$

Rate at which sales are changing on Jan 10 $= S'(10)$

$$= \frac{-18}{11^3}$$

$$\approx -0.0135 = \boxed{-\$13.50/\text{day}}$$

b.) From Example 6, $S'(2) \approx -0.667$

On Jan 2, sales are falling \$667/day

Rate of change on Jan 10 is still negative on Jan 10, but much smaller than on Jan 2. Sales are <u>stabilizing</u>

63.) Dept at end of 1999 $= D(4)$

$$= 4.95 + 0.402(4) - 0.1067(4)^2 + 0.0124(4)^3 - 0.00024(4)^4$$

≈ 5.583 trillion dollars

$$D'(x) = 0.402 - 0.1067(2)x + 0.0124(3)x^2 - 0.00024(4)x^3$$

$$= 0.402 - 0.2134x + 0.0372x^2 - 0.00096x^3$$

Rate of change of debt in 1999 $= D'(4)$

$$= 0.402 - 0.2134(4) + 0.0372(4)^2 - 0.00096(4)^3$$

$$\approx 0.0822$$

$= 82.2$ billion / year

64.a) Debt in 2003 $= D(8)$

≈ 6.703 trillion dollars

Debt in 2001 $= D(6)$

≈ 5.888 trillion dollars

Debt is NOT twice as big in 2003

b) Rate of increase in 2003 $= D'(8)$

≈ 0.584 trillion / year

Rate of increase in 2001 $= D'(6)$

≈ 0.253 trillion / year

Debt is increasing more than twice as fast in 2003 vs 2001