HW 9 – MA 131

Section 3.1

34.) $y' = 4(x^2-1)^3(2x)(x^2+1)^5 + (x^2-1)^4 5(x^2+1)^4(2x)$

$= 8x(x^2-1)^3(x^2+1)^5 + 10x(x^2-1)^4(x^2+1)^4$

$= (x^2-1)^3(x^2+1)^4\left[8x(x^2+1) + 10x(x^2-1)\right]$

$= (x^2-1)^3(x^2+1)^4(8x^3+8x+10x^3-10x)$

$= (x^2-1)^3(x^2+1)^4(x)(18x^2-2)$

$x = \pm 1$ Never 0 $x = 0$

$0 = 18x^2 - 2$

$2 = 18x^2$

$\frac{1}{9} = x^2$

$\pm\frac{1}{3} = x$

$x = -1$

$y = (1-1)^4(1+1)^5 = 0$

$x = -1/3$

$y = (1/9 - 1)^4(1/9+1)^5 \approx 1.06$

$x = 0$

$y = (0-1)^4(0+1)^5 = 1$

$x = 1/3$

$y = (1/9 - 1)^4(1/9+1)^5 \approx 1.06$

$x = 1$

$y = (1-1)^4(1+1)^5 = 0$

Minima: $(-1, 0), (0, 1), (1, 0)$

Maxima: $(-1/3, 1.06), (1/3, 1.06)$

55.) Use Quotient Rule

$$y' = \frac{(1+.25x^2)10 - 10x(.5x)}{(1+.25x^2)^2}$$

$$0 = 10 + 2.5x^2 - 5x^2 = 10 - 2.5x^2$$

$$2.5x^2 = 10$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

$$y = \frac{10 \cdot 2}{1 + .25 \cdot 4} = \frac{20}{1+1} = \frac{20}{2} = 10$$

$\boxed{\text{Maximum at } (2, 10)}$

67.) Use Quotient Rule

$$\boxed{b'(t) = \frac{[h(t)]^2 w'(t) - w(t) \cdot 2h(t) \cdot h'(t)}{[h(t)]^4}}$$

68. a) $b = \dfrac{w(t)}{[h(t)]^2} = \dfrac{50}{1.55^2} \approx \boxed{20.8}$

Not overweight or at risk of being overweight

b) $b'(12) = \dfrac{(1.55)^2\, 7 - 50 \cdot 2 \cdot 1.55 \cdot .05}{(1.55)^4}$

$\boxed{\approx 1.57}$

c) $b(13) \approx b(12) + b'(12) \approx 20.8 + 1.57 = 22.3$

Section 4.5

1.) $y' = \dfrac{2}{2x} = \dfrac{1}{x}$

2.) $y' = \dfrac{1}{2x}$

3.) $y' = \dfrac{1}{x+3}$

4.) $y' = \dfrac{12x - 3}{6x^2 - 3x + 1}$

6.) $y' = \dfrac{2x e^{x^2+2}}{e^{x^2+2}} = 2x$

12.) Use product rule

$$y' = \frac{\ln 2x\,(1/x) - \ln x\,(1/x)}{[\ln 2x]^2}$$

17.) $\frac{d}{dx}\left(\frac{x-1}{x+1}\right) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2}$

$$= \frac{2}{(x+1)^2}$$

$$y' = \frac{\frac{2}{(x+1)^2}}{\frac{x-1}{x+1}} = \frac{2}{(x+1)^2}\,\frac{x+1}{x-1} = \frac{2}{(x+1)(x-1)}$$

27.) $f'(x) = \frac{\sqrt{x}\,(1/x) - \ln x\,(1/2\sqrt{x})}{x}$

$$0 = \frac{\sqrt{x}}{x} - \frac{\ln x}{2\sqrt{x}} = \frac{1}{\sqrt{x}} - \frac{\ln x}{2\sqrt{x}}$$

$$2\sqrt{x}\,\frac{\ln x}{2\sqrt{x}} = \frac{1}{\sqrt{x}}2\sqrt{x}$$

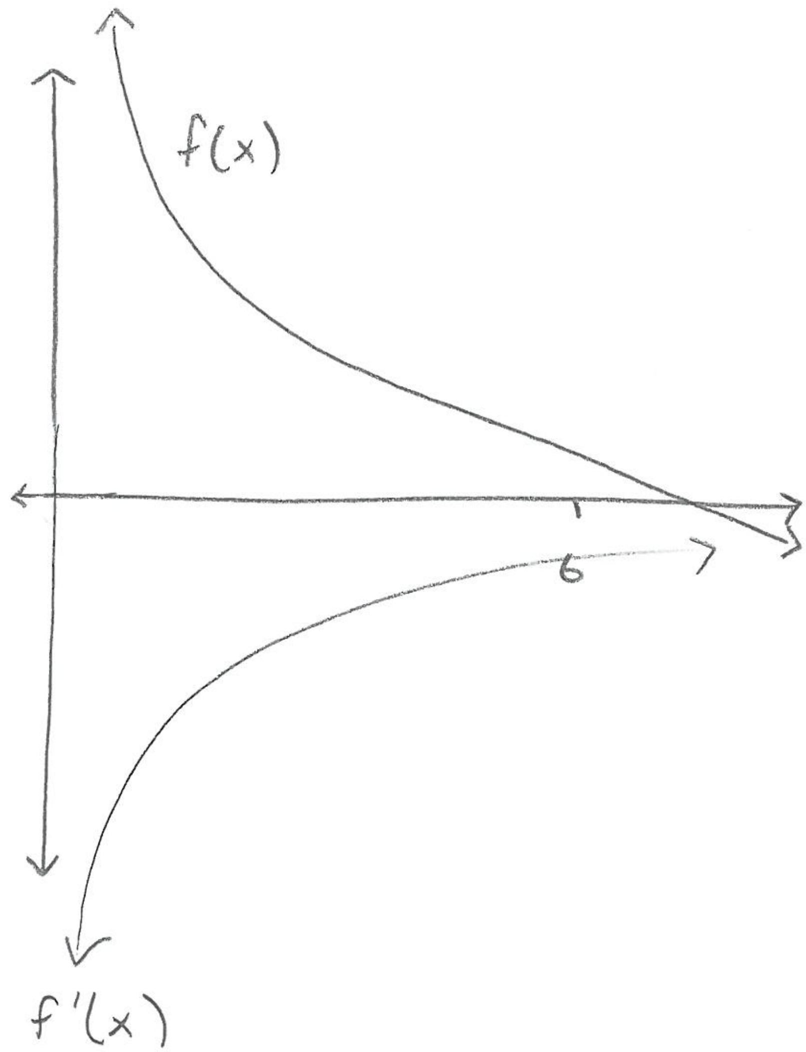
$$\ln x = 2$$

$$x = e^2$$

$$f(e^2) = \frac{\ln e^2}{\sqrt{e^2}} = \frac{2}{e}$$

$$\boxed{\text{Max at } (e^2, 2/e)}$$

38. a) $f'(x) = \frac{-14.09}{x}$



b) $f(3.25) = 26.48 - 14.09\ln(3.25) \approx 9.9$

c) $15 = 26.48 - 14.09\ln x$

$11.48 = 14.09\ln x$

$.815 = \ln x$

$e^{.815} = x$

$2.26 = x$

$\boxed{2.26\%}$

d) $f'(2.75) = -\frac{14.09}{2.75} = -5.12 \frac{\text{bites}}{\text{\% concentration}}$

e) $-10 = \frac{-14.09}{x}$

$-10x = -14.09$

$\boxed{x = 1.41\%}$

Section 5.1

2. a) $P(0) = 10{,}000$

b) $P(t) = 10{,}000e^{.55t}$

c) $P(5) = 10{,}000e^{.55 \cdot 5} \approx 156{,}426$

d) $.55$

e) $P'(100{,}000) = .55 \cdot 100{,}000 = 55{,}000 \frac{\text{bacteria}}{\text{hour}}$

f) $34{,}000 = .55\, P$

$\boxed{61{,}818 \approx P}$

4. a) 300

b) $P'(t) = .01\, P(t)$

c) $600 = 300e^{.01t}$

$2 = e^{.01t}$

$\ln 2 = .01t$

$\frac{\ln 2}{.01} = t$

$\boxed{t \approx 69.3 \text{ days}}$

d) $1200 = 300e^{.01t}$

$4 = e^{.01t}$

$\boxed{t \approx 138 \text{ days}}$

$\ln 4 = .01t$

$\frac{\ln 4}{.01} = t$

12. a) $P(1974) \approx 9$ million

b) $P(1978) = 10$ million, $\boxed{1978}$

c) $P'(1974) = .025\, P(1974) = .025 \cdot 9$

$= .225 \frac{\text{million people}}{\text{year}}$

d) $.275 = .025\, P(t)$

$11 = P(t)$

$\boxed{t \approx 1981}$

14. a) $P(t) = 12e^{-.00043t}$

b) 12

c) $-.00043$

d) $P(943) = 12e^{-.00043 \cdot 943} \approx 8$

f) $-.004 = -.00043\, P$

$P \approx 9.3$

g) $P(1612) = 6$

$P(3224) = 3$

$P(4836) = 1.5$

26) $.91 C_0 = C_0 e^{-.00012t}$

$.91 = e^{-.00012t}$

$\ln .91 = -.00012t$

$$\frac{\ln .91}{-.00012} = t$$

$t \approx 786$ years

Not King Arthur's since its only 786 years old