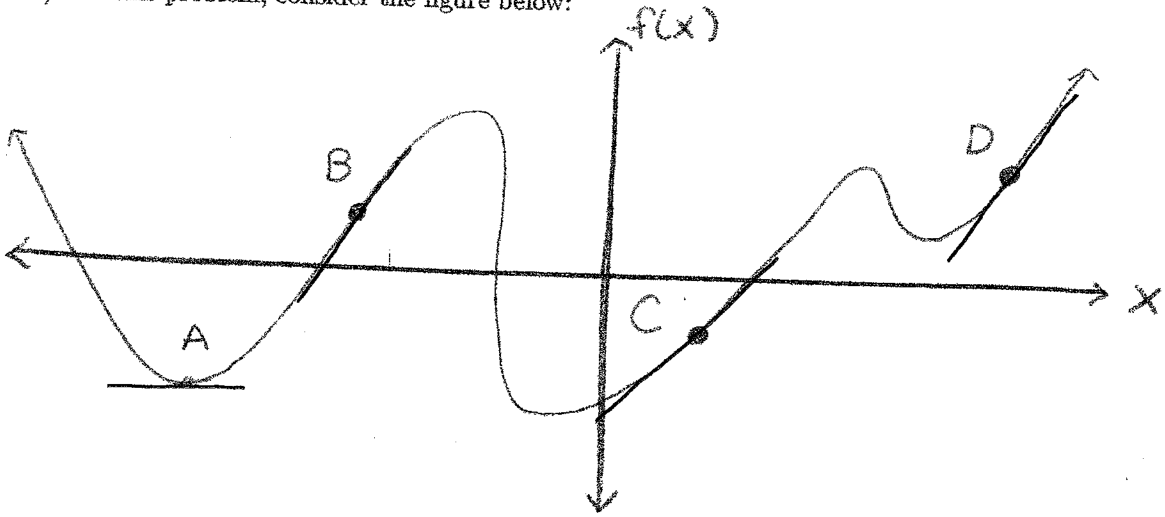
Name: ______________________________

MA 131 Test 1 Form A

1.) For this problem, consider the figure below:



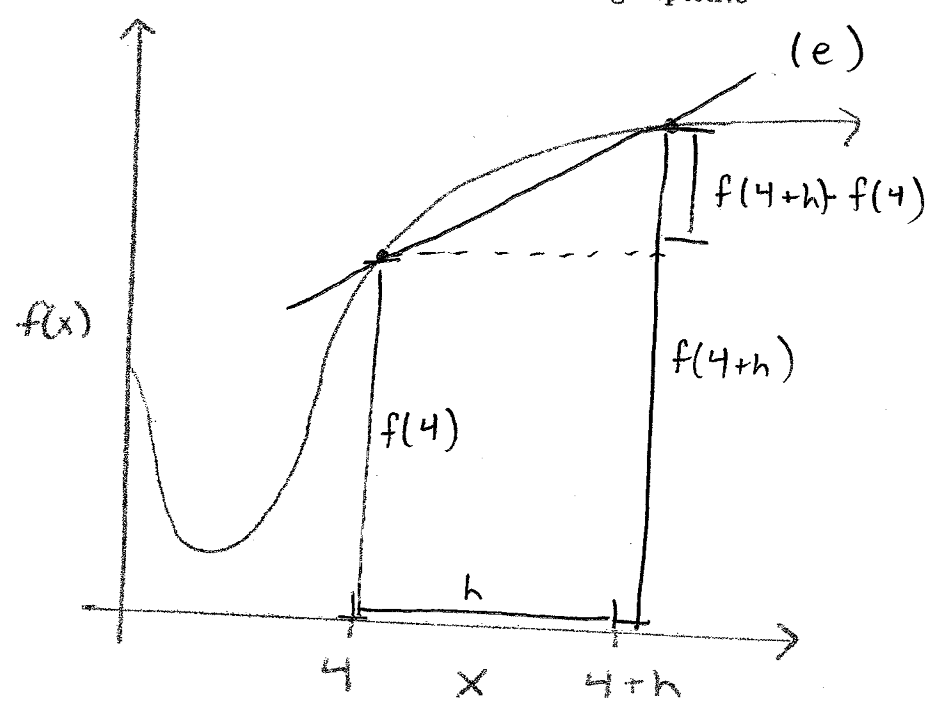
(a) Draw tangent lines at the points A, B, C, and D.

(b) At each point, is $f'(x)$ positive, negative, or approximately 0?

A. ______ 0 ______
B. ______ + ______
C. ______ + ______
D. ______ + ______

2.) Below is the graph of a function $f(x)$. Draw and *clearly label* 5 line segments of the following respective lengths((a) through (e))

(a) h
(b) $f(4+h)$
(c) $f(4)$
(d) $f(4+h) - f(4)$
(e) Draw a line with slope $\dfrac{f(4+h)-f(4)}{h}$

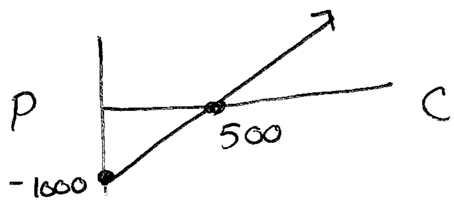


3.) Suppose an ice cream store has \$1000 of fixed overhead expenses per month (rent, insurance, etc.), and that they make \$2 in profit for every ice cream cone they sell.

(a) Write an equation relating the store's total profit (P) and the number of ice cream cones they sell in a month (C)

$$P = 2C - 1000$$

(b) Graph the equation in part (a).



(c) Suppose that the store sells 1,387 ice cream cones in June. The owner can sell 30 *more* cones in July if they spend \$40 on advertising. Would it be a good decision for the owner to spend this \$40 on advertising? Why?

$$\frac{\text{Change in Profit}}{\text{Change in Cones}} = 2$$

Change in Profit $= 2 \cdot 30 = 60$

$60 - 40 = 20$

4.) Consider some function $f(x)$.

(a) State the limit definition of the derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) State the definition of the derivative that involves tangent lines:

$f'(x)$ is slope of tangent line at x

(c) In plain English, explain what the derivative of $f(x)$ is:

rate of change of $f(x)$ at x

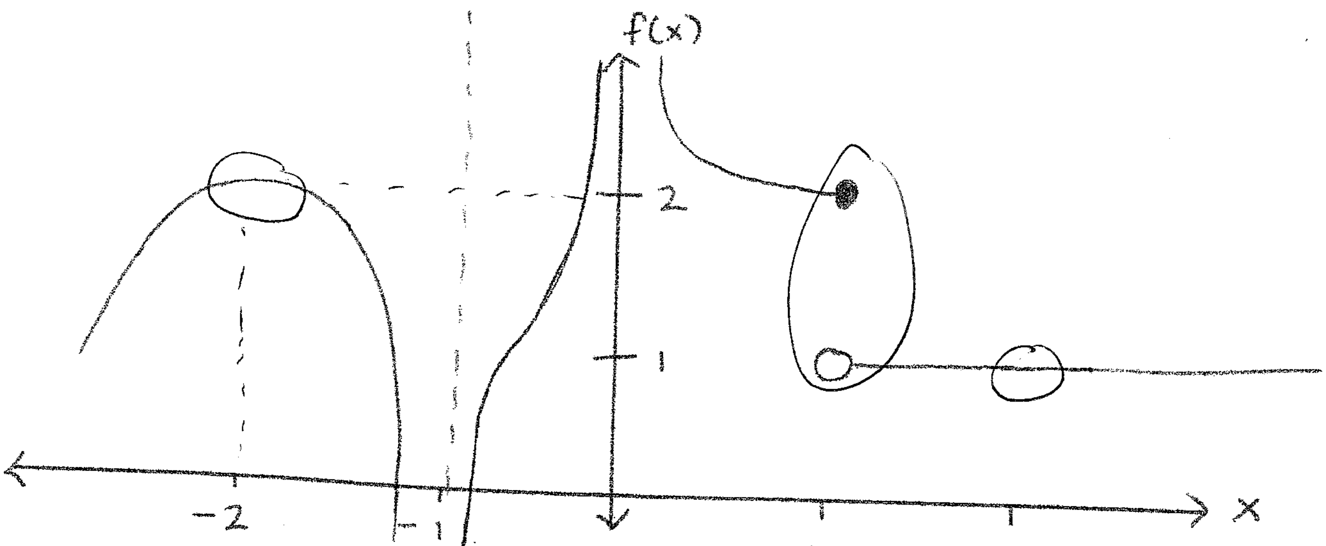
5.) Evaluate the following limits:

(a) $\lim_{x\to 2} \frac{x^2-4}{x-2} =$ $= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2} (x+2) = 4$

(b) $\lim_{x\to 3} \frac{x^2-4}{x-2} =$ $\frac{3^2-4}{3-2} = \frac{9-4}{1} = \frac{5}{1} = 5$

(c) Below is the graph of a function $f(x)$. Use the graph to determine the following 3 limits (if they exist):

$\lim_{x\to -2} f(x) =$ 2 $\qquad \lim_{x\to 1} f(x) =$ DNE $\qquad \lim_{x\to 2} f(x) =$ 1



6.) Compute derivatives of the following 4 functions

(a) $f(x) = 4x + \frac{3}{x}$

$= 4x + 3x^{-1}$

$f'(x) = 4 + 3(-1)x^{-2}$

$= 4 - \frac{3}{x^2}$

(b) $f(x) = 7 + 8x + 2x^2 + x^3$

$f'(x) = 8 + 4x + 3x^2$

(c) $f(x) = \frac{1}{2x^2+5x+3}$

$= (2x^2+5x+3)^{-1}$

$f'(x) = -1(2x^2+5x+3)^{-2}(4x+5)$

(d) $f(x) = 4$

$f'(x) = 0$

7.) A standardized test prep company found that the average student's score on a test (s) is related to the time they spent studying (t) by the following function:

$$s(t) = 400 + 30\sqrt{t}$$

(a) What would the average student score if they studied 1 hour? 9 hours? 100 hours?

$s(1) = 400 + 30\sqrt{1} = 430$

$s(9) = 400 + 30\sqrt{9} = 400 + 30 \cdot 3 = 490$

$s(100) = 400 + 30\sqrt{100} = 400 + 30 \cdot 10 = 700$

(b) What was the average rate of score increase for the first 9 hours of study?

$$\frac{s(9) - s(0)}{9 - 0} = \frac{490 - 400}{9} = \frac{90}{9} = 10$$

(c) Find $s'(1)$ and $s'(9)$, and write a sentence explaining what each of these two numbers means.

$$s'(t) = 30\left(\tfrac{1}{2}\right)t^{-1/2} = 15t^{-1/2} = \frac{15}{t^{1/2}} = \frac{15}{\sqrt{t}}$$

$s'(1) = 15/\sqrt{1} = 15$

$s'(9) = 15/\sqrt{9} = 15/3 = 5$

At 1 hour of study, score is increasing 15 points/hour

s(t)