Name: ______________________

MA 131 Test 1 Form B

1.) Consider some function $f(x)$.

(a) State the limit definition of the derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

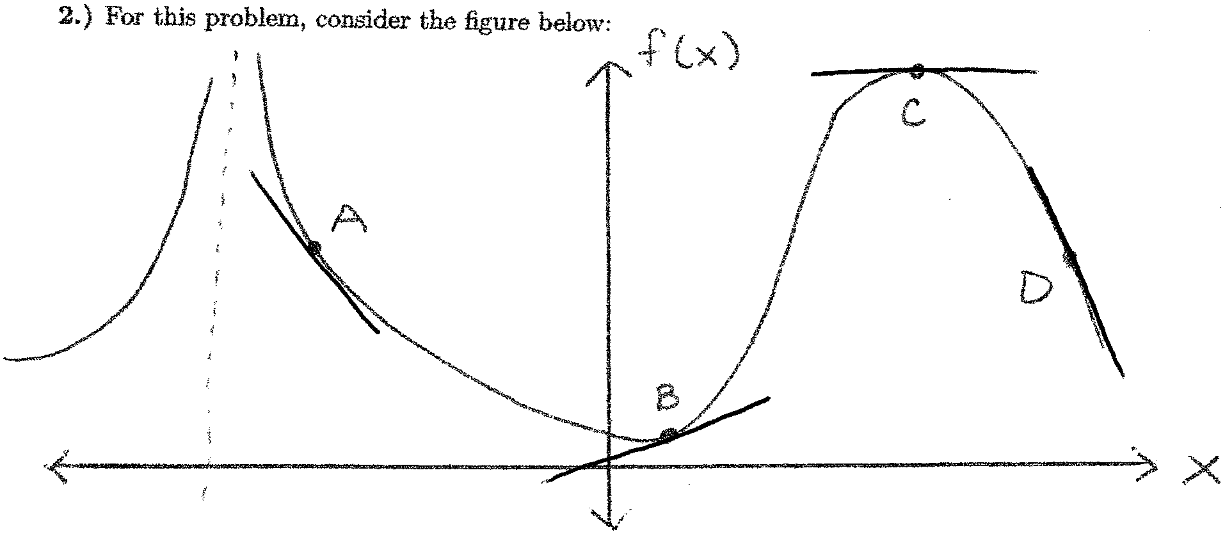
(b) State the definition of the derivative that involves tangent lines:

The derivative of $f(x)$ at x, written $f'(x)$, is the slope of the tangent line to $f(x)$ at x

(c) In plain English, explain what the derivative of $f(x)$ is:

The rate of change of $f(x)$ at X

2.) For this problem, consider the figure below:



(a) Draw tangent lines at the points A, B, C, and D.

(b) At each point, is $f'(x)$ positive, negative, or approximately 0?

A. −
B. +
C. 0
D. −

3.) Below is the graph of a function $f(x)$. Draw and *clearly label* 5 line segments of the following respective lengths((a) through (e))

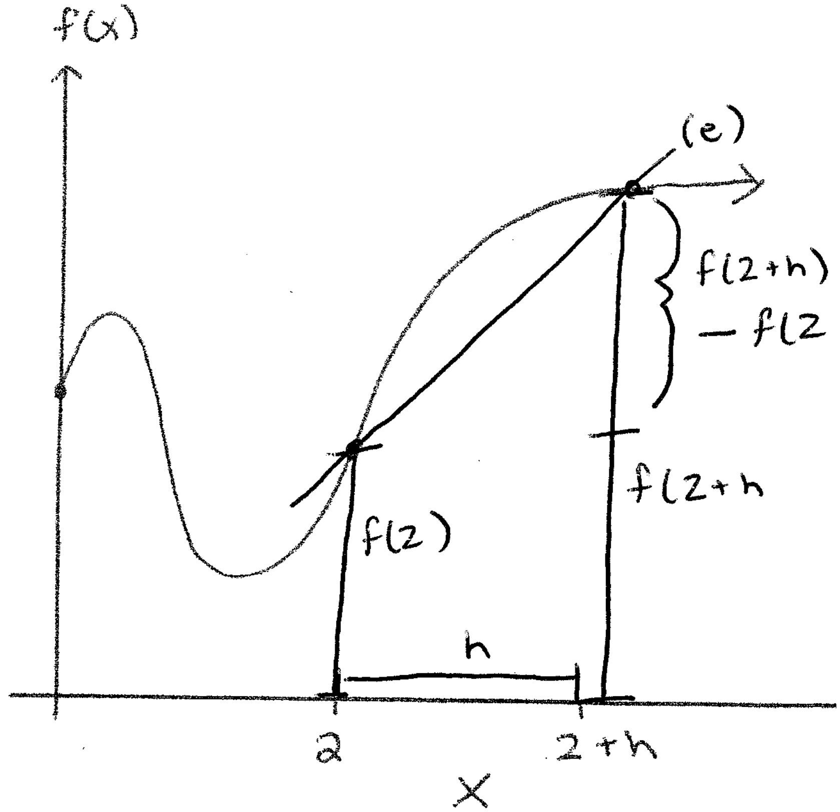
(a) h

(b) $f(2+h)$

(c) $f(2)$

(d) $f(2+h)$ $- f(2)$

(e) Draw a line with slope $\frac{f(2+h)-f(2)}{h}$



4.) Compute derivatives of the following 4 functions

(a) $f(x) = 2x + \frac{7}{x}$

$= 2x + 7x^{-1}$

$f'(x) = 2 + 7(-1)x^{-2}$

$= 2 - \frac{7}{x^2}$

(b) $f(x) = 3 + 6x + 4x^2 + 4^3$

$f'(x) = 6 + 4 \cdot 2 \cdot x$

$= 6 + 8x$

(c) $f(x) = \frac{1}{3x^2+x+8}$

$= (3x^2 + x + 8)^{-1}$

$f'(x) = -(3x^2+x+8)^{-2}(6x+1)$

$= -\frac{6x+1}{(3x^2+x+8)^2}$

(d) $f(x) = 19$

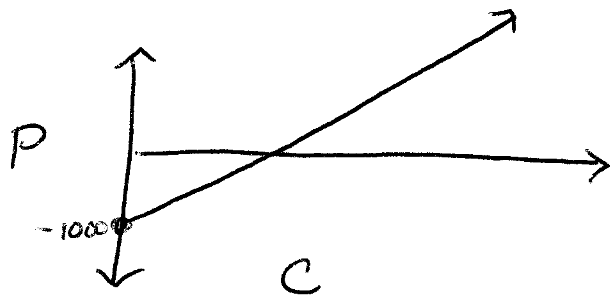
$f'(x) = 0$

5.) Suppose an ice cream store has $1000 of fixed overhead expenses per month (rent, insurance, etc.), and that they make $1 in profit for every ice cream cone they sell.

(a) Write an equation relating the store's total profit (P) and the number of ice cream cones they sell in a month (C)

$$P = C - 1000$$

(b) Graph the equation in part (a).



(c) Suppose that the store sells 1,387 ice cream cones in June. The owner can sell 30 *more* cones in July if they spend $40 on advertising. Would it be a good decision for the owner to spend this $40 on advertising? Why?

The 30 more cones adds $30 in profit, which is not enough to offset the $40 advertising expense. Dont buy the advertising

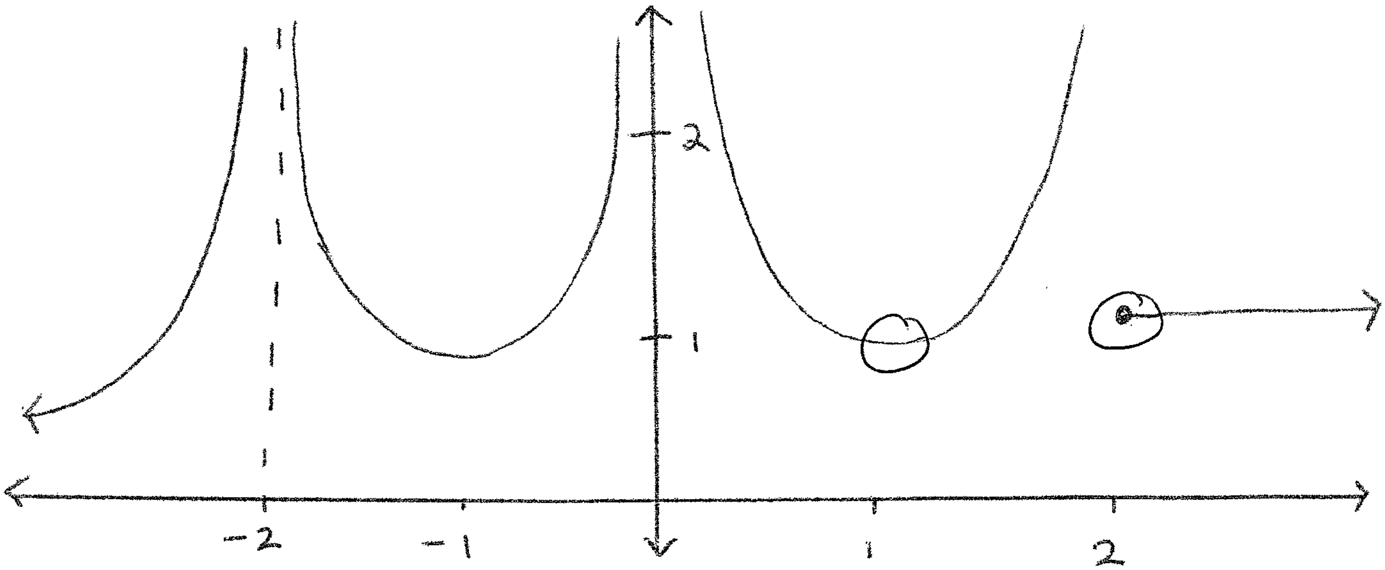
6.) Evaluate the following limits:

(a) $\lim_{x\to 3} \frac{x^2-4}{x-2} = \frac{3^2-4}{3-2} = \frac{9-4}{1} = 5$

(b) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} (x+2) = 4$

(c) Below is the graph of a function $f(x)$. Use the graph to determine the following 3 limits (if they exist):

$\lim_{x\to-2} f(x) =$ DNE $\qquad \lim_{x\to1} f(x) =$ 1 $\qquad \lim_{x\to2} f(x) =$ DNE



7.) A standardized test prep company found that the average student's score on a test (s) is related to the time they spent studying (t) by the following function:

$$s(t) = 400 + 30\sqrt{t}$$

(a) What would the average student score if they studied 1 hour? 4 hours? 100 hours?

$s(1) = 400 + 30\sqrt{1} = 430$

$s(4) = 400 + 30\sqrt{4} = 460$

$s(100) = 400 + 30\sqrt{100} = 700$

(b) What was the average rate of score increase for the first 4 hours of study?

$$\frac{s(4) - s(0)}{4 - 0} = \frac{460 - 400}{4} = \frac{60}{4} = 15 \ \frac{\text{points}}{\text{hr of study}}$$

(c) Find $s'(1)$ and $s'(4)$, and write a sentence explaining what each of these two numbers means.

$s'(t) = \frac{15}{\sqrt{t}}$

At 1 hour of study, score is increasing at a rate of 15 pts/hr.

$s'(1) = \frac{15}{1} = 15$

$s'(4) = \frac{15}{\sqrt{4}} = 15/2 = 7.5$